

Duality Properties of the Hardy–Littlewood Maximal Operator in Spaces Defined on Locally Compact Abelian Groups

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Annotation: One of the central problems in harmonic analysis is the study of the boundedness of the Hardy–Littlewood maximal operator in function spaces.

In 2005, Lerner proved that this operator is bounded on a variable-exponent reflexive Lebesgue space if and only if it is bounded on its dual space.

When studying harmonic analysis on locally compact Abelian groups (LCAGs), certain difficulties arise because standard notions such as “kernel” or “cube” are not available. In this context, their role is replaced by nested families of compact subgroups, which allow the construction of maximal and singular operators.

This work investigates various boundedness properties of the Hardy–Littlewood maximal operator on LCAGs. The operator is defined using a collection of nested, exhaustive subgroups. Furthermore, the conditions for boundedness on the dual space are studied, yielding a result analogous to Lerner’s theorem in the setting of locally compact Abelian groups.

References

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